$a_r = \ddot{r} - r(\dot{\theta})^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

(9) $\quad v_r = \dot{r} \qquad v_\theta = r\dot{\theta}$

$$a = \frac{d}{dt}(\dot{r}\hat{r}) + \frac{d}{dt}(r\dot{\theta}\hat{s})$$

$$= \dot{r}\frac{d\hat{r}}{dt} + \hat{r}\ddot{r} + r\dot{\theta}\frac{d\hat{s}}{dt} + r\ddot{\theta}\hat{s} + \dot{r}\dot{\theta}\hat{s}$$

$$= \dot{r}(\hat{s}\dot{\theta}) + \hat{r}\ddot{r} + r\dot{\theta}(-\hat{r}\dot{\theta}) + r\ddot{\theta}\hat{s} + \dot{r}\dot{\theta}\hat{s}$$

$$= (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{s} + [\ddot{r} - r(\dot{\theta})^2]\hat{r}$$

$$a = (\ddot{r} - r(\dot{\theta})^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{s}$$

So Radial Component of acceleration $= a_r = \ddot{r} - r(\dot{\theta})^2$

And Transverse " " " $= a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

Example:- A particle P moves in a plane in such a way that at any time t, its distance from a fixed point O is $r = at + bt^2$ and the line Connecting O and P makes an angles $\theta = ct^{3/2}$ with (Connecting 'O' and 'P' makes angle) a fixed line OA. Find the radial and transverse Components of the velocity and acceleration of the particle at $t = 1$.

Sol:- $\quad r = at + bt^2 \longrightarrow (1)$

$\theta = ct^{3/2} \longrightarrow (2)$

from (1) $\quad \frac{dr}{dt} = \dot{r} = a + 2bt$

$\frac{d^2r}{dt^2} = \ddot{r} = 2b$

from (2) $\quad \frac{d\theta}{dt} = \dot{\theta} = \frac{3}{2}c\sqrt{t}$

$\frac{d^2\theta}{dt^2} = \ddot{\theta} = \frac{3}{4}\frac{c}{\sqrt{t}}$

Now at $t = 1$

$\dot{r} = a + 2b \quad , \quad \ddot{r} = 2b \quad$, and $r = a + b$

and $\dot{\theta} = \frac{3}{2}c \quad , \quad \ddot{\theta} = \frac{3}{4}c$

As we know that

$$a = [\ddot{r} - r(\dot{\theta})^2]\hat{r} + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{s}$$