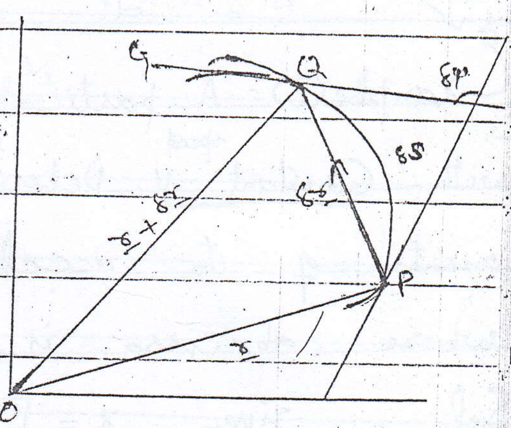
As $\hat{t}\cdot\hat{t} = 1\cdot 1 = 1$ $\quad \because \hat{t}\,\hat{t} = 1$

$\hat{t}\cdot\hat{t} = 1$ Diff w.r.t. t

$$\frac{d\hat{t}}{dt}\cdot\hat{t} + \hat{t}\cdot\frac{d\hat{t}}{dt} = 0$$

$$\hat{t}\,\frac{d\hat{t}}{dt} = 0$$

Show $\frac{d\hat{t}}{dt}$ $\perp$r to tangent line since it will be along normal line. Let $\hat{n}$ be the unit vector along the normal line.



$$\frac{d\hat{t}}{dt} = \lim_{\delta t\to 0}\left|\frac{\delta\hat{t}}{\delta t}\right|$$

By Chain rule

$$\frac{\delta\hat{t}}{\delta t} = \frac{\delta\hat{t}}{\delta s}\cdot\frac{\delta s}{\delta\psi}\cdot\frac{\delta\psi}{\delta t}$$

$$\lim_{\substack{\delta t\to 0\\ \delta s\to 0\\ \delta\psi\to 0}}\left|\frac{\delta\hat{t}}{\delta s}\cdot\frac{\delta s}{\delta\psi}\cdot\frac{\delta\psi}{\delta t}\right|$$

$$\Rightarrow \lim_{\delta t\to 0}\left|\frac{\delta s}{\delta t}\right|\cdot\lim_{\delta s\to 0}\left|\frac{\delta\psi}{\delta s}\right|\quad\lim_{\delta\psi\to 0}\left|\frac{\delta\hat{t}}{\delta\psi}\right|$$

$$= \frac{ds}{dt}\cdot\frac{d\psi}{ds}\cdot 1$$

$$= v\cdot\frac{d\psi}{ds}$$

$$\left|\frac{d\hat{t}}{dt}\right| = \frac{v}{\rho} \qquad \because \frac{d\psi}{ds} = \frac{1}{\rho}\ \text{(radius of curvature)}$$

$$\left|\frac{d\hat{t}}{dt}\right| = \frac{v}{\rho}\hat{n} \quad \text{Put in (2)}$$

$$\boxed{\underline{a} = \frac{dv}{dt}\hat{t} + \frac{v^2}{\rho}\hat{n}}$$ Tangential Component of acceleration $a_t = \frac{dv}{dt}$ and Normal Component of acceleration $a_n = \frac{v^2}{\rho}$.